Comparison of Wing Strength of Kelly D and Hatz

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In the wake of last year's fatal Kelly D accident at Poplar Grove, speculations came up about the structural integrity of the design. Since I had already performed a structural analysis of the Hatz biplane [1], I offered to take a look at the Kelly D structure. I received a set of plans soon thereafter and went to work. A vacation, moving to a new home and various other commitments kept me from getting results any sooner.

The whole analysis was performed using SI units and these are in general also used in this report. A conversion table between SI and American units is given at the end.

The analysis starts with the definition of the flight envelope (see also [1]). The airplane performance and specifications (table 1) together with the requirements

Wing span	26 ft	7.9 m
Wing area	202.8 ft^2	18.8 m^2
Gross weight	1600 lbs	725 kg
Top speed	127 mph	204 km/h
Cruise speed	90 mph	145 km/h
Stall speed	45 mph	72.4 km/h

Table 1: Specification of Kelly D biplane

of FAR Part 23 [2] yield the flight envelope shown in figure 1. The maximum positive maneuvering load factor is assumed to be 4.4 g. It can be seen that for point C of the flight envelope the load factor is higher (4.68 g). This value is the result of the gust load requirements of FAR 23. As we will see later this loadcase is the most critical one for the wing structure.

For all points (SACDEFG) of the flight envelope the loads in the structure must be calculated. The lift distribution is shown in figure 2 together with the one for the Hatz [1]. The loads and moments in all the wing compo-

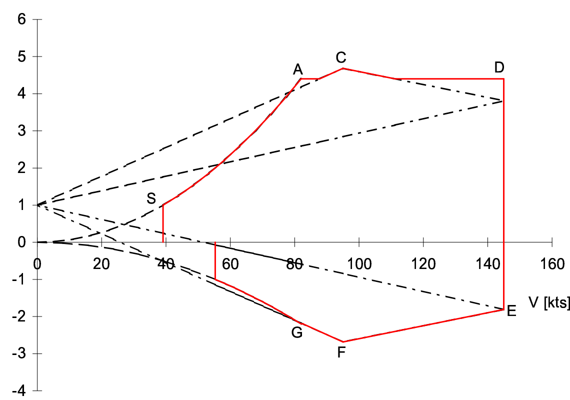

Figure 1: Flight envelope for Kelly D biplane

nents (spars, wires, interplane struts,...) are then calculated for all loadcases. Figure 3 shows the bending moments in the upper front and rear spars for loadcase C. This loadcase is found to be the most critical for the dimensioning of the wing spars (this is also true for the Hatz). The critical section of the spar is just inboard of the interplane strut-attachment where the maximum bending moment is superposed to a compression load F_N due to the flying wires. The numerical values of the maximal bending moment M_b and the normal force F_N are given in table 2.

Even though the Kelly D and the Hatz are similar in size and performance the loads in the spars differ, the maximum in the front spar being considerably higher for the Kelly D. This is due in part to the higher maximal g-force (n=4.68 versus n=4.56) but mainly to the differences in the geometry (wing spans, no center sec-

[1] During the present analysis an error in the lift distribution originally used for the Hatz analysis was discovered. For critical components the corrected distribution yields generally lower loads. The wrong distribution was thus conservative and the conclusions drawn at the time are still valid.

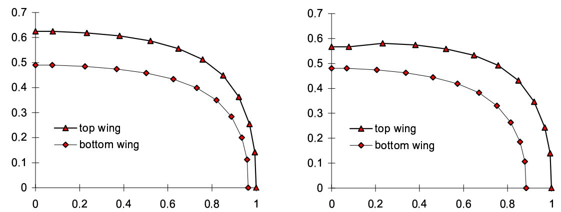

Figure 2: Lift distribution on top and lower wing of KellyD (left) and Hatz (right)

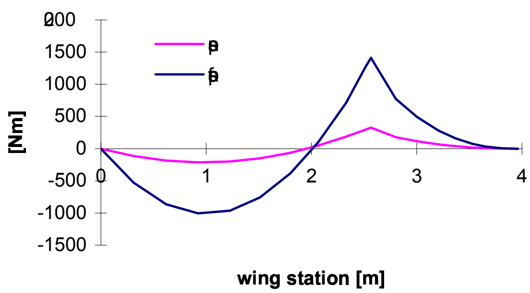

Figure 3: Bending moment distribution on front and rear spar of Kelly D wing for loadcase C

tion for the Kelly D).

From a structural point of view the maximum stress in a given cross section of the spar is of interest. This is obtained by the following formula:

$$\sigma_{\max} = \frac{F_N}{A} + \frac{M_b}{I} \cdot \frac{h}{2}$$

F_N is the load in the direction of the spar, A the cross section of the spar, M_b the bending moment, I the moment of inertia and h the height of the spar. The values of A and I for the Kelly D and the Hatz front spars are given in figure 4.

Even though the loads which have to be carried by the Kelly D front spar are higher than those of the Hatz,

loadcaseC	KellyD	Hatz
bending moment (Nm)	1411	979
normal force (N)	8202	6233

Table 2: Maximum bending moments and normal forces in front spar of upper wing for the Kelly D and Hatz biplanes

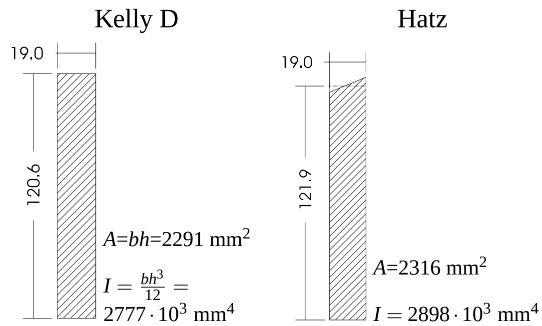

Figure 4: Geometry of front spars. Kelly D at left and Hatz at right

the spar is almost the same size, actually even slightly smaller. The maximum stress in the critical section is:

Kelly D

$$\sigma_{\max} = \frac{8202}{2291} + \frac{1411 \cdot 10^3}{2777 \cdot 10^3} \cdot \frac{120.6}{2} = 34.2 \text{ N/mm}^2$$

Hatz

$$\sigma_{\max} = \frac{6233}{2316} + \frac{979 \cdot 10^3}{2898 \cdot 10^3} \cdot \frac{121.9}{2} = 23.3 \text{ N/mm}^2$$

These numbers must now be compared to the allowable stress values in the wood. These are found for example in ANC-18 [3]. For spruce with a 15% moisture content we get the values shown in table 3. FAR 23 requires the structure to support the limit loads (the maximum loads expected) without permanent deformation. Therefore the stress in the spars must be compared to the value of F_{bp}. We define the safety margin for limit loads as:

Fiber stress at proportionality limit	F_{bp}	=5300 psi = 36.5 N/mm^2
Modulus of rupture	F_{bu}	=9400 psi = 64.8 N/mm^2

Table 3: Allowable stress in spruce spars with 15% moisture content under bending loads

$$MS_{\text{limit}} = \frac{F_{bp}}{\sigma_{\max}} - 1$$

A negative value of MS indicates an insufficiently strong structure. The ultimate load is obtained by multiplying the limit load by a factor of 1.5. According to the regulations the structure must be able to withstand ultimate loads without failure for 3 seconds (FAR23.305b).

Therefore the margin of safety for ultimate loads is:

$$MS_{\text{ultimate}} = \frac{F_{bu}}{1.5 \cdot \sigma_{\text{max}}} - 1$$

The safety margins thus obtained are given in table 4. Although all values of MS are positive, this is just barely

	MS_{limit}	MS_{ultimate}	loadcase
Kelly D	0.06	0.26	C (n=4.68)
Hatz	0.57	0.85	C (n=4.56)

Table 4: Safety margins for top front spar at critical section (inboard of interplane strut)

true in the case of MS_{limit} for the Kelly D. If one further considers the variability of the wood itself (ANC-18 lists a decreased value of F_{bp}=4200 psi = 28.9 N/mm^2 for spruce with 20% moisture content) and other uncertainties, a value of $MS_{\text{limit}} = 0.06$ is in my opinion clearly insufficient.

The analysis didn't take into account the plywood doublers which are present in the area and contribute to the strength of the spar. On the other hand, the additional weakening due to the bolt holes was not considered either.

What does all this mean in practice? The spar of the Kelly D built per plans using good quality wood and workmanship will just barely satisfy the structural requirements of FAR 23. For airplanes already built, I would therefore strongly suggest:

- not to perform any aerobatic maneuvers

- to limit the maximum take-off weight to 1400 lbs instead of 1600 lbs. In that case the safety margin becomes $MS_{\text{limit}} = 0.2$

For new airplanes the following structural improvements are suggested:

- Use stronger wood for the upper front spars. Douglas fir is on the average about 10% stronger (F_{bp}=5900 psi = 40 N/mm^2) than spruce. Keeping the dimensions of the spar the same, this yields a safety margin of $MS_{\text{limit}} = 40/34.2 - 1 = 0.17$.

- The preferred solution would be to increase the height of the front spar (fig. 5). Since the original design already has ribs which are joined at the spar (not like the Hatz where a capstrip runs over the spar), this modification could be implemented without too much trouble. The maximum stress and the safety margin are then:

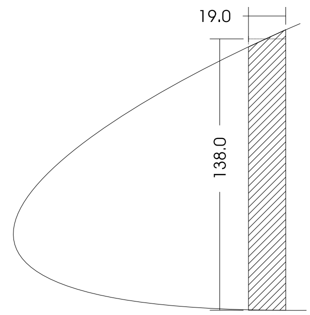

Figure 5: Strengthened Kelly D top front spar

$$\sigma_{\text{max}} = \frac{8202}{2630} + \frac{1411 \cdot 10^3}{4161 \cdot 10^3} \cdot \frac{138}{2} = 26.6 \text{ N/mm}^2$$
$$MS_{\text{limit}} = 0.26$$

For either solution I would also extend the 1/4" plywood doublers on bay inboard and outboard. It is important to use high quality 5-ply birch plywood and to bevel it generously. The face grain of the plywood should run parallel to the spar.

References

[1] J.Müller; 'Structural analysis of the Hatz Airplanes', AHA Newsletter Vol. 9, No. 4, December 2000

[2] Federal Aviation Regulations, Part 23, 'Airworthiness Standards: Normal, Utility, and Aerobatic Category Aircraft', Federal Aviation Administration

[3] anonymous; 'Design of Wood Aircraft Structures'; ANC-18 Bulletin, Governement Printing Office, June 1951

Conversion table

multiply	by	to obtain
Newton [N]	0.2248	pounds force [lbf]
meter [m]	3.281	feet [ft]
inch [in]	25.4	millimeter [mm]
Nm	0.7376	lbf·ft
N/mm^2	145.03	psi
mph	1.6093	km/h